Grade 3

Number Talks

Purposes:

To develop computational fluency (accuracy, efficiency, flexibility) in order to focus students' attention so they will move from:

- figuring out the answers any way they can to . . .
- becoming more efficient at figuring out answers to . . .
- just knowing or using efficient strategies

Description:

The teacher gives the class an equation to solve mentally. Students may use pencil and paper to keep track of the steps as they do the mental calculations. Students' strategies are shared and discussed to help all students think more flexibly as they work with numbers and operations.

Materials:

- Prepared problems to be explored
- Chalkboard, white board, or overhead transparency
- Individual white boards or pencil and paper
- Optional: Interlocking cubes and/or base ten materials

Time: 15 minutes maximum

Directions:

Example: 98 + 87

1. Write an expression **horizontally** on the board (e.g. 98 + 87).

2. Ask students to think first and estimate their answer before attempting to solve the problem. Post estimates on the board. This will allow you to see how the students are developing their number sense and operational sense.

3. Ask students to mentally find the solution using a strategy that makes sense to them. Encourage students to "think first" and then check with models, if needed. Have tools available to help students visualize the problem if they need them (e.g. interlocking cubes, base ten blocks).

4. Ask students to explain to a partner how they solved the problem.

5. While students are discussing their strategies, walk among the groups listening to the explanations. Find those strategies you want to call attention to for the whole class. Choose strategies for discussion that you might want other students to think about and possibly experiment with. In the example 98 + 87, you might want to call attention to any of the following strategies:

   87 – 2 = 85
   98 + 2 = 100
   100 + 2 = 185

   98 is almost 100 which is a friendly number
   100 + 87 = 187
   187 – 2 = 185

   90 + 80 = 170
   8 + 7 = 15
   170 + 15 = 185

6. Call on a student to fully explain the steps he/she followed to solve the problem.

7. Record the steps precisely as the student explains them to you. Ask clarifying questions as needed to be sure you understand the flow of the student's thinking process. Be sure to be explicit about the mathematics. For example:

   - "Why does this strategy work?"
   - "Will this strategy always work?" How do you know?"
   - "What did you know about the number 87 that allowed you to do that?"
   - "Why did you need to subtract 2?"
   - "Where did the 90 come from? The 80? The 8? The 7?"

8. As time allows, ask other students to share different methods they used for solving the equation. Follow up on each strategy shared by asking similar questions to those included in step 7. Record these methods.

9. It is very important to facilitate a discussion about how the different representations/strategies relate to each other and result in the same answer.

Example - Guiding the Share-Out:

Scenario 1: 63 – 27

Public Recording	Student's Explanation
63 + 3 = 66 27 + 3 = 30 66 – 30 = 36	"I added the same amount to both numbers to keep the difference the same. I chose 3 because it makes the 27 a "friendly" number to subtract."

Possible teacher response

"You said you added 3 to both numbers. How does adding 3 to both numbers keep the difference the same? How can you convince me? How can you show that?"

Scenario 2: 63 - 27

Public Recording	Student's Explanation
27 + **3** = 30 30 + **30** = 60 60 + **3** = 63 **3 + 30 + 3** = 36	"I added 3 to 27 to make 30. I added 30 more to make 60. Then I added 3 more to make 63. I added together all the numbers I used to get to 63."

Possible teacher response:

"So you used, an "adding up" strategy. How does adding numbers help to find a difference? What did you know about subtraction that made you think of adding the numbers you did? How did you keep track of what numbers to add?

Scenario 3: 63 - 27

Public Recording	Student's Explanation
63 – 20 = 43 7 = 3 + 4 43 – 3 = 40 40 – 4 = 36	"I subtracted 20 from 63 and that made 43. I broke 7 into 3 + 4 and subtracted the 3 from the 43. That made 40. I still had to take 4 away from the 40, so the answer is 36."

Possible teacher response:

"Why did you begin by subtracting 20? Where did the 20 come from? Why did you break the 7 up into a 3 and 4? Why did you subtract the 3 before you subtracted the 4? Why do you think all of these strategies work to find the same answer?"

Scaffold:

- When beginning number talks, make sure that the problems and quantities are accessible and within each child's zone of proximal development. The numbers must be accessible so that the students are solving the equations mentally.

- As the students' flexibility, accuracy and efficiency improve, increase the rigor of the problems by adjusting the numbers or operations. Allow the students to document on paper their intermediate steps **as** they are solving the problem.

- If you have students in your classroom who are performing at diverse instructional levels, select 3 different problems for students to solve at 3 different levels. Give students the choice of which problem they will solve. Select problems with different size numbers so that all students have access to a problem and all students are working at a level that pushes them to their optimal level. For example:

  463 – 27 63 – 27 63 – 7

- As you begin to introduce the multiplication module, include number talks focusing on strategies that help students making meaning of the multiplication facts. For example: 9 x 3

  9 + 9 + 9 = 27
  10 + 10 + 10 – 3 = 27
  (10 x 3) – 3 = 27
  (4 x 3) + (5 x 3) = 27
  (2 x 9) + 9 = 27

Test Prep:

Some children who understand many mathematical ideas do not fare well on a standardized test given in a multiple-choice format. Often, children guess a "letter" rather than reasoning through the problem. To improve children's test taking strategies while building number and operational sense, the following strategies are suggested:

- Pose a problem just as a problem would be posed with a "Number Talk." For example:

  A company has 6 big trucks. Each truck has 18 wheels. How many wheels is this in all.

  Ask students to think about the problem in a way that makes sense to them.

- Only after the children have thought about the problem, show them an A., B., C., and D. response. Ask them to choose the answer that is closest to their thinking. For example:

  A. 24
  B. 96
  C. 108
  D. 116

- Ask students to publicly share the methods they used for solving the problem. For example:

"I know that the total is 6 groups of 18. I know that 18 is 10 + 8. Six groups of ten are 60. Six groups of eights are the same as two eights three times [(8 + 8) + (8 + 8) + (8 + 8)]. 16 + 16 + 16 = 30 + 12 + 6 = 48. 48 plus 60 equals 108. There are 108 wheels in all. The answer is C.

or

- *"I know that 6 times 10 equals 60. I know that 6 times 8 equals 48. 48 plus 60 equal 108. There are 108 wheels in all. The answer is C.*

- When it fits the problem, facilitate conversations about the reasonableness of each choice (e.g. *"How did you know which operation to use? Which choices could you have eliminated immediately? Why? Why would A not have been a reasonable choice? What number would have to be in the one's place? Why?*).

- As students "guess" less and "reason" more, pose the problem along with the A, B, C, and D responses. Ask students to think about the problem in a way that makes sense to them and then select the closest answer to their thinking. Ask students to share with a partner which responses they would eliminate immediately and why.

- The important piece is that students take the time to think and reason about the problem before they choose an answer (or guess). This must be a "habit of mind" whenever they are confronted with a problem to solve. Using this format once a week beginning very early in the school year could help students "break" the habit of guessing and assist in higher scores on standardized tests.

<u>**Examples**</u>:

87 + 49	37 + 86	58 –39
91 – 53	370 + 99	499 + 76
864 – 500	104 – 39	372 + 98
855 – 56	750 + 250	359 + 36
100 – 49	156 – 38	462 – 33
1200 – 49	7200 – 49	1156 – 38
9956 - 38	2462 – 33	10,462 – 33
3 x 15	11 x 9	9 x 52

Notes about Number Talks:

A. Keep them short.

B. Encourage sharing and clarify students' thinking

C. Teach intentionally

- Start where your children are.
- Choose related sequences of problems.
- Focus students' thinking:

  See if you can . . .
  How many will there be if . . . ?
  What if . . . ?
  Can you use what you know about the last problem to help you think about this problem?

- Encourage students to "think first" and then check with models, if needed.
- Chart the students' thinking so that it can be saved and referred to later.

D. Create a safe and supportive environment

- Accept answers without praise or criticism.
- Allow students to ask questions of each other.
- Encourage students to listen to each other.
- Encourage students to self-correct.

E. Vary the Number Talks to meet the range of needs.

- Vary the sharing strategies used.

  Pair share
  Share whole group
  Explain someone else's strategy

- Vary the level of difficulty within a number talk.

  Use written problems
  Use smaller numbers
  Use story problems

- Record the students' thinking using correct notation on the board, on the overhead, or on chart paper.

F. Give students lots of practice with the same kinds of problems.

G. When planning or implementing a Number Talk, consider the following:

- How do students get their answers?
- Can students use what they know for related problems?
- How well can students verbalize their thinking?
- Are errors way off or are they reasonable?

H. The role of the teacher during a number talk is to facilitate and guide the conversation.

- The teacher purposefully chooses children to share strategies that will move the class toward computational fluency.
- The teacher asks questions that draw attention to the relationships among strategies.
- **It is important to focus on the mathematics, not just the variety of strategies.**

Mathematically, why does the strategy work?

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Mathematically, why does the strategy work?

Grade 3

Number Strings

Purpose:

- To use number relationships to solve problems and to learn number facts
- To use known facts and relationships to determine unknown facts
- To develop and test conjectures
- To make generalizations about mathematical relationships, operations and properties

Description:

This routine focuses on developing a sense of pattern and relationships among related problems. The task is at a higher level than merely recalling basic facts. Students identify and describe number patterns and relationships within and among equations. Students make conjectures about the patterns and relationships they notice. During this process, students explain their reasoning. Over time, students develop generalizations about important number relationships, operations and properties. These generalizations assist in solving problems and learning number facts.

Materials:

- Prepared list of number strings
- Whiteboard, chart paper, or overhead transparencies
- Student journals, student whiteboards, or scratch paper

Time: 15 minutes maximum per session

Directions:

Example:
a. 3 x 2 =
b. 3 x 4 =
c. 3 x 8 =
d. 3 x 16 =
e. 3 x 32 =
f. 3 x 64 =

1. Write equation "a" and ask students to solve mentally (e.g., 3 x 2 =).
   Equation "a" should be easily accessible to all students.
2. Have students check their answer with a partner.
3. Ask one student to share his/her solution with the class. Write the answer on the board to complete the equation (3 x 2 = 6).
4. Students show thumbs up or thumbs down for agreement or disagreement.
   - If there is agreement, go to equation "b."
   - If there is disagreement, facilitate a class conversation around the strategies the student(s) used to arrive at the answer. Allow students to revise answers.

5. Give the students problem "b" to solve mentally (e.g., 3 x 4). Repeat, #'s 2, 3, and 4 above.

6. Write problem "c" (e.g., 3 x 8). Ask students how they could use what they know about the first two equations to solve this equation. Partner talk.

7. A volunteer shares his/her mathematical reasoning that derived an answer to this equation.

   Note: If students are having difficulty sharing relationships, ask questions such as the following:
   - *How are equations "a" and "b" alike?*
   - *How are equations "a" and "b" different?*
   - *Describe the relationship between the factors?*
   - *Describe the relationship between the products?*
   - *How can we use these relationships to predict the product for equation "c?"*

8. Write problem "d" (e.g., 3 x 16). Ask students to predict their answer to this problem. Students share their predictions with their partner and explain their thinking. Teacher writes predictions on the board.

9. A volunteer shares his/her mathematical reasoning that derived the answer to this equation.

10. Repeat steps 8 and 9 for equations "e" and "f."

11. When the string is completed, facilitate a conversation around how relating a known equation can help students solve unknown equations. Listen for what relationships students notice throughout the string and how students are able to extend patterns beyond the string you have written. Ask students to make statements about the patterns and/or relationships that helped them to complete the string.

12. Examine the "conjectures" that the students share. Ask questions such as:
    - *Will doubling one factor always result in a doubled product? How can you prove your conjecture?*
    - *Will this always work? How can you prove your conjecture?*

13. **Do not <u>tell</u> students the generalization. Ask students to make conjectures first and then ask them to test their conjectures using three or more examples. If the conjecture always holds true, then the students can make "generalizations."**

<u>Variation 1</u>:

In multiplication, many strings begin by doubling one factor while leaving the other factor constant (e.g. 3 x 2 becomes 3 x 4). As one factor is doubled, so is the product (e.g., 3 x <u>2</u> = <u>6</u>; therefore 3 x <u>4</u> = <u>12</u>).

Generalization: When one factor is multiplied by a particular amount, the product will be multiplied by the same amount.

Examples:

a. 3 x 2 =	a. 7 x 2 =	a. 2 x 3 =
b. 3 x 4 =	b. 7 x 4 =	b. 2 x 6 =
c. 3 x 8 =	c. 7 x 8 =	c. 2 x 12 =
d. 3 x 16 =	d. 7 x 16 =	d. 2 x 24 =
e. 3 x 32 =	e. 7 x 32 =	e. 2 x 48 =
f. 3 x 64 =	f. 7 x 64 =	f. 2 x 96 =

Variation 2:

Sometimes, the pattern is predictable because a factor is being doubled over and over, and the product doubles over and over. But then, the pattern may change and the numbers and products cease to double.

Generalizations: Numbers are the sum of more than one quantity (e.g., 12 = 8 + 4). The distributive property states that when a number is being multiplied by a particular factor, it is equivalent to multiplying the number by the parts that make up that factor [e.g., 3 x 12 = (3 x 4) + (3 x 8); or 3 x 12 = (3 x 10) + (3 x 2)].

Examples:

a. 3 x 2 =	a. 4 x 1 =	a. 6 x 1 =
b. 3 x 4 =	b. 4 x 2 =	b. 6 x 2 =
c. 3 x 8 =	c. 4 x 4 =	c. 6 x 4 =
d. 3 x 10 =	d. 4 x 8 =	d. 6 x 8 =
e. 3 x 12 =	e. 4 x 12 =	e. 6 x 12 =
f. 3 x 6 =	f. 4 x 13 =	f. 6 x 13 =
g. 3 x 14 =	g. 4 x 15 =	g. 6 x 15 =

Using Strings to Learn Multiplication Facts

Strings can be helpful to assist students to learn their multiplication facts as they learn to see the relationships among the facts.

Example I: If a student cannot remember 8 x 6, but knows 4 x 6, all the student has to do is double the product of 4 x 6 because 8 = 2 x 4.

4 x 6 = 24
8 x 6 = 48

Example II: If a student cannot remember 8 x 6, but knows 2 x 6 and 6 x 6, all the student has to do is find the product of these two equations and then find the sum of the products because 8 = 2 + 6.

2 x 6 = 12   6 x 6 = 36   8 x 6 = 48

Guiding Questions:

- What pattern(s) do you see?
- What stayed the same? What changed?
- How did it change?
- How did knowing the answers to the first equation help you figure out the answer to the next equation?
- Does this always work? How do you know?
- How are equations "a" and "b" alike?
- How are equations "a" and "b" different?
- What is the relationship between the factors?
- What is the relationship between the products?
- How can we use these relationships to predict the product for equation "c?"

Grade 3

Number of the Day

Purpose:

- To help develop students' flexibility with numbers and operations
- To develop understanding of number composition and part-whole relationships
- To explore equivalent arithmetic expressions

Description:

For "Number of the Day", students write equations that equal the number of days they have been in school. Students generate ways of combining numbers and operations to make that number.

Materials:

- Chart paper
- Individual white boards or journals

Time: 15 minutes maximum

Directions:

1. Post the chart paper.

2. Write the *Number of the Day* at the top of the chart paper.

3. Ask students to tell you everything they know about the number (e.g., 24; the number of sodas in four six packs; the number of eggs in two dozen; the number of crayons in a box of crayons; the number of classes at school; two tens and four ones; one cent less than a quarter).

4. Ask students to think of several models and equations that would represent the *Number of the Day.*

5. Ask students to represent the *Number of the Day* in at least four or more different ways.

6. Have students document these in their daily math journals.

7. Observe the students as they work and purposefully choose students to share whose representations will move the class towards further development of number and operational sense.

8. Strategically call on those students who represented the number in meaningful ways that you would like to highlight.

9. Write those representations on the chart paper as students dictate.

10. Finish by leading a class conversation around those representations that best connect to concepts recently learned. Be purposeful about the mathematics. Help students make mathematical connections whenever possible.

Example: The Number of the Day is "12"

6 + 6 = 12	2 x 2 x 3 = 12
12 = 22 – 10	1/2 of 24 = 12
3 x 4 = 12	24 ÷ 2 = 12
12 = 10 + 10 – 8	11 1/2 + 1/2 = 12
10 + 2 = 12	12 = 3 x 3 + 3
2 x 12 – 12 = 12	1 ten and two ones

Constraints

When students are familiar with the structure of *Number of the Day*, connect it to the number work they are doing in particular units. Add constraints to the equations to practice and reinforce different mathematical concepts. Ask students to include:

- Both addition and subtraction
- Three numbers
- Combinations of 10
- Multiplication
- Division
- Expanded notation
- Doubles
- Doubles plus one
- Specific multiples
- Associative/Distributive/ Commutative Properties of Addition/Multiplication
- Zero property
- Order property
- Draw a number line and correctly place the number
- Write equations with answer and equal sign on the left (45 = 15 + 20 + 10)
- Represent *number of the day* with manipulatives
- Represent *number of the day* with stories and pictures
- Represent *number of the day* with money
- Include fractions
- Emphasize using tens
- Emphasize using hundreds

Grade 3

In and Out Boxes

Purpose:

Students recognize, describe, and generalize patterns and functional relationships. A function is a relationship in which two sets are linked by a rule that pairs each element of the first set with exactly one element of the second set.

Description:

Students analyze a set of number pairs to determine the rule that relates the numbers in each pair. The data are presented in the form of a function table (T-table) generated from an "In and Out Box." Students will describe rules for relating inputs and outputs and construct inverse operation rules.

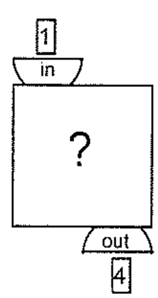
Materials:

- T-table transparencies (What is the Rule?)
- "In and Out Box" transparency

Time: 15 minutes maximum

Directions:

1. Display the "In and Out Box" on the overhead. Tell students that when you put a number in the box, a secret rule changes the number, and out comes a new number. Demonstrate with "1." You put "1" into the box, and "out" comes "4."



in	out

2. We organize the inputs and outputs with a T-table.

3. The job of the students is to determine the rule.

4. Write the first number in each column (e.g., 1 in the first column, 4 in the second column).

   Note: The example that we will be using may be complicated for early in the year. You may want to use a different "rule" when you introduce this routine to your class.

in	out
1	4

5. Ask pairs of students to discuss possible rules that could cause the number in the first column to become the number in the second column (e.g., add 3, multiply by 4, multiply by 2 and add 2).

6. Write the second number in each column (e.g., "2" in the first column, "6" in the second column).

in	out
1	4
2	6

7. Ask the students to check the rule they came up with for the first pair of numbers to see if the rule will apply to both sets of numbers. If the rule does not apply, ask students to think about a rule that could describe both sets of numbers.

8. Give students a third pair of numbers (e.g., 3 in the first column, 8 in the second). Does the rule still work? If not, students think about a different rule that could describe all three sets of numbers.

in	out
1	4
2	6
3	8

9. Ask students to write/discuss with their partner another pair of numbers that could fit their rule.

10. Ask for volunteers to share what they think other pairs of numbers could be that would fit the rule. Record these numbers on the transparency (without judgment as to whether it is correct or incorrect).

in	out
1	4
2	6
3	8
5	14
10	22
4	11

11. Partners discuss all the pairs of numbers on the transparency to see if they agree, or if there are inconsistencies in the pattern. Facilitate a class discussion about what operation(s) were working on each "in" to get each "out."

For Example:

> For the teacher. DO NOT TELL

$1 + 1 + 2 = 4$
$2 \times 1 + 2 = 4$
$2 + 2 + 2 = 6$
$2 \times 2 + 2 = 6$
$3 + 3 + 2 = 8$
$2 \times 3 + 2 = 8$

in	out
1	4
2	6
3	8
~~5~~	~~14~~
10	22
~~4~~	~~11~~

Discuss similarities and differences, such as "Sue multiplied the first number by 2 and then added 2 more. But Julian added 1 to the first number then multiplied **that** number by 2. Why do these two rules still work?"

12. Based on the above conversation for each of the specific numbers, have partners discuss what they believe the rule is for changing the first number in the pair to the second number in the pair. <u>Students are now speaking in general terms, what must be done to any number, not just the numbers written on the table</u>.

    For the teacher. DO NOT TELL. In the example, each number in the "in" box is added to itself, plus two more; or, doubled, plus two more; or, multiplied by two, plus two more.

13. Ask for volunteers to share their rules. Record on the transparency the rules that students have generated. You will be recording the words, not numeric symbols (e.g., "The number in the 'out' box is equal to each number in the 'in' box added to itself, plus two more").

14. Discuss how the rules are the same. Discuss how the rules are different. Do the rules accurately apply to each of the pairs of numbers? Facilitate a class discussion about how the rules work and how each is a different representation of the same pattern.

15. Once the class agrees on rules that will work, ask the partners to figure out what other pairs of numbers would follow the rules.

16. Ask students to volunteer some of their pairs as you record them. Have students explain why each pair of numbers follows the rule.

17. Give the students new "in" numbers. What would be the "out." How do you know?

<u>Scaffold:</u>

- Use smaller numbers and/or rules with only 1 operation (e.g., multiply by 2).

Extensions:

- Include rules with more than one step.
  (e.g., multiply by 2, then subtract 1)

- Give the students the first pair of numbers as explained above. Then, give the "out" number in the set. Ask students to find the "in" number in the set.
  Note: Working backwards provides opportunities for using inverse operations to find missing elements and writing rules. For example: The In-Out rule is +4; the Out-In rule is –4.

in	out
1	5
3	7
	9

Examples:

In	Out
1	3
2	4
3	5
4	

In	Out
1	2
2	4
3	6
5	
	12

In	Out
1	0
4	3
7	6
11	
	15

In	Out
2	7
3	8
9	14
20	
	36

In	Out
3	15
6	30
5	25
10	
	20

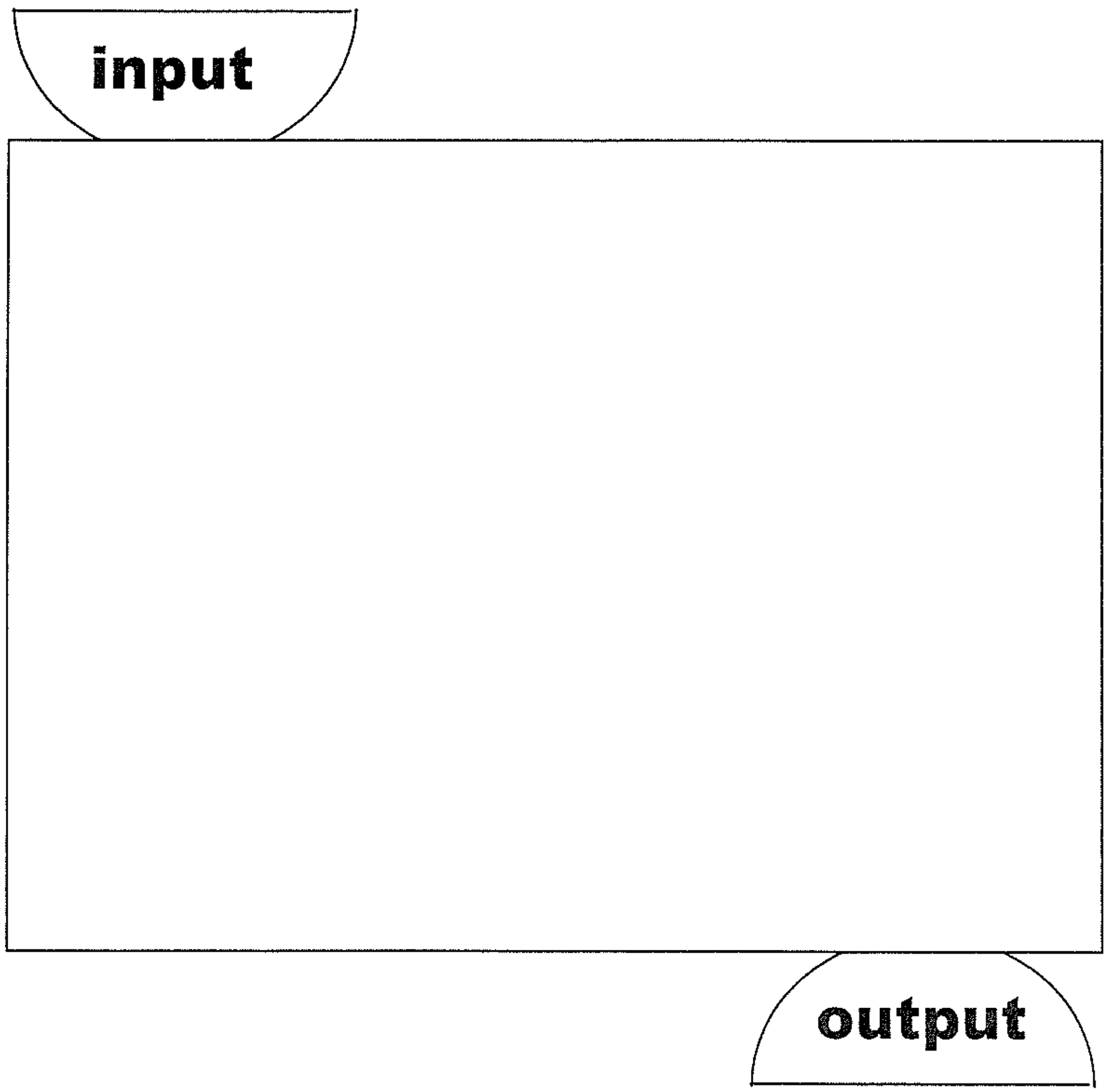
In	Out
1	3
5	11
10	21
8	
	7

In	Out
15	25
4	14
7	
	63

In	Out
4	12
6	18
9	27
3	
	30

In	Out
3	12
6	15
11	20
30	
	28

In and Out Box



In	Out

What is the rule?

Grade 3

Concepts of Equality

Purpose:

- To develop an appropriate conception of equality and the equal sign
- To develop understanding that the equal sign denotes the relation between two equal quantities (rather than a command to carry out a calculation)

Description:

- Students are engaged in a discussion about the meaning of the equal sign.
- The context of this discussion is true/false and open number sentences.
- The number sentences provide a focus for students to articulate their ideas and to challenge their conceptions.
- The discussions assist in developing ways of thinking and communicating that embody the principles of algebraic reasoning.
- Students articulate mathematical principles that often are not explored or stated.
- Students must justify the principles that they propose in ways that convince others, and they must recognize and resolve conflicting assumptions and conclusions,

Materials:

Purposely planned number sentences and open number sentences. The numbers selected should be easily accessible to students. The focus is on the meaning of the equal sign, not on practice of operations.

Time: 15 minutes maximum

Pre-assessment

Before beginning this series of routines, ask your students to complete the following on a half-sheet of paper:

What number would you put in the box to make this a true number sentence?

$$8 + 4 = \square + 5$$

This information is for you. Do not discuss this problem with your students.

This problem was given to thirty typical elementary-grade classes. The responses were as follows:

	Response - Percent Responding			
	7	12	17	12 and 17
Grades 1 and 2	5%	58%	13%	8%
Grades 3 and 4	9%	49%	25%	10%
Grades 5 and 6	2%	76%	21%	2%

From: Falkner, Levi, & Carpenter, 1999

This data suggests that many elementary school students have serious misconceptions about the meaning of the equal sign as a relation between two equal quantities. Many seem to interpret the equal sign as a command to carry out a calculation (the answer is…).

This misconception limits students' ability to learn basic arithmetic ideas with understanding and their flexibility in representing and using those ideas. This creates even more serious problems as they move to algebra.

Directions:

1. Engage students in a general discussion about true/false number sentences or what it means for a number sentence to be true or false. Provide examples asking whether the number sentence is true or false and how they know it is true or false. For example:

   8 – 5 = 3   2 x 2 = 5   99 + 68 = 167

2. Once students are familiar with true/false number sentences, equations can be introduced that may encourage them to examine their conceptions of the meaning of the equal sign. Pose one equation at a time and lead a discussion as to whether the equation is true or false. Students must justify their claims. Do not tell. Lead a discussion and ask questions.

   For example:

4 + 5 = 9	9 = 4 + 5	9 = 9
4 + 5 = 4 + 5	4 + 5 = 5 + 4	4 + 5 = 6 + 3
2 x 4 = 8	8 = 2 x 4	8 = 8
2 x 4 = 2 x 4	2 x 4 = 4 x 2	2 x 4 = 1 x 8
15 – 7 = 8	8 = 15 – 7	8 = 8
15 – 7 = 15 – 7	15 – 7 = 7 - 15	15 – 7 = 16 – 8

10 ÷ 2 = 5	5 = 10 ÷ 2	5 = 5
10 ÷ 2 = 10 ÷ 2	10 ÷ 2 = 2 ÷ 10	10 ÷ 2 = 15 ÷ 3

Many of the examples above do not follow the familiar form with two numbers and an operation to the left of the equal sign and the answer to the right of the equal sign. This may throw some students into disequilibrium. Asking students to choose whether each number sentence is true or false can encourage them to examine their assumptions about the equal sign.

Note: We are trying to help students understand that the equal sign signifies a relation between two numbers. It is sometimes useful to use words that express that relation more directly (e.g., "Nine is the same as 4 plus 5").

3. Including zero in a number sentence may encourage students to accept a number sentence in which more than one number appears after the equal sign. For example:

   9 + 5 = 14   9 + 5 = 14 + 0   9 + 5 = 0 + 14   9 + 5 = 13 + 1

4. Open number sentences given after the corresponding true/false questions are a nice way to follow up on the ideas that came out of the discussion of the true/false number sentence. The question being asked is:

   "What number can you put in the box to make this number sentence true?"

4 + 5 = □	9 = 4 + □	9 = □
4 + 5 = □ + 5	4 + 5 = □ + 4	4 + 5 = □ + 3
□ = 4 + 5	4 + □ = 9	
2 x 4 = □	8 = 2 x □	8 = □
2 x 4 = □ x 4	2 x 4 = □ x 2	2 x 4 = □ x 8
□ = 2 x 4	2 x □ = 8	
15 – 7 = □	8 = 15 – □	8 = □
15 – 7 = □ – 7	15 – 7 = □ – 8	□ = 15 – 7
15 - □ = 8		
10 ÷ 2 = □	5 = 10 ÷ □	5 = □
10 ÷ 2 = □ ÷ 2	10 ÷ 2 = □ ÷ 3	□ = 10 ÷ 2
10 ÷ □ = 5		

<u>Scaffolding:</u>

The following are benchmarks to work toward as children's conception of the equal sign evolves.

1. Getting children to be specific about what they think the equal sign means (even if their thinking is incorrect). To do this, the conversation must go beyond just comparing the different answers to the problem. For example, in the problem 8 + 4 = □ + 5, some children might say:

   The equal sign must be preceded by two numbers joined by a plus or a minus and followed by the answer (resulting in an answer of 12 to this problem).

   You have to use all the numbers (resulting in an answer of 17 to this problem).

   Though this understanding is not correct, the articulation of conceptions represents progress.

2. Children accept as true some of the number sentences that are not of the form a + b = c (e.g., 8 = 5 + 3; 8 = 8; 3 + 5 = 8 + 0; or 3 + 5 = 3 + 5).

3. Children recognize that the equal sign represents a relation between two equal numbers (rather than "the answer is"). Children might compare the two sides of the equal sign by carrying out the calculation on each side.

4. Children are able to compare the mathematical expression without actually carrying out the calculation. For example: 8 + 4 = □ + 5

A child might say, "*I saw that the 5 over here (pointing to the 5 in the number sentence) was one more than the 4 over here (pointing to the 4 in the number sentence), so the number in the box had to be one less than the 8. So it's 7.*"

<u>Guiding Questions:</u>

- *Why do you think that?*
- *Why do you think that you cannot write number sentences that look like that?*
- *Do you agree or disagree with Student A? Why?*
- *Why do you believe this equation is true?*
- *Why do you believe this equation is false?*
- *What do you do when there is more than one number that follows the equal sign?*
- *How do you know that the number that you put in the box makes the number sentence true?*
- *How can you figure out the number that goes in the box without doing any calculation?*

<u>Reference</u>

Carpenter, Thomas P. Franke, Megan Loef, Levi, Linda, <u>Thinking Mathematically: Integrating Arithmetic & Algebra in Elementary School</u>, Portsmouth, N.H.: Heinemann, 2003.

Falkner, Karen P., Levi, Linda, & Carpenter, Thomas P. 1999. "Children's Understanding of Equality: A Foundation for Algebra." *Teaching Children Mathematics 6*, 232-236.

Grade 3

Thinking Relationally

Purpose:

- To make the learning of arithmetic richer
- To think flexibly about mathematical operations
- To compare mathematical expressions without actually carrying out the calculation
- To help students recognize without having to calculate that the expressions on each side of the equal sign represent the same number
- To provide a foundation for smoothing the transition to algebra

  Note: In algebra, students must deal with expressions that involve adding, subtracting, multiplying, and dividing but that are not amenable to calculation (e.g., $3x + 7y - 4z$). They have to think about relations between expressions ($5x + 34 = 79 - 2x$) as they attempt to figure out how to transform equations in order to solve them.

Description:

Students are engaged in conversations about the relationships between numbers and how these relationships can be useful in finding solutions to problems. Students analyze expressions through the context of true/false and open number sentences. Students find ways to solve the problems by using number relations before calculating the answers.

Materials:

Purposely planned equations.

Note: Select equations that cannot be easily calculated. We want students to be motivated to look for relations. If equations can be easily calculated, the need does not exist to look for number relations.

Time: 15 minutes maximum

Directions:

1. To successfully implement relational thinking routines, the following classroom norms must be established:

- Students explain their thinking
- Students listen to one another
- Alternative strategies for solving a given problem are valued and discussed
- Solutions that involve more than simply calculating answers are not only accepted, but valued

2. You, the teacher, will need to make decisions based on the needs of your class. As you select problems:

    - Start with relatively easy problems and selected problems that provide an appropriate level of challenge based on what you have observed students doing on previous problems.
    - Select problems that will challenge students but not be too difficult for them.
    - Make decisions about what problems to use next based on students' responses to problems that they had already solved.

3. **Goal 1: For students to recognize that they do not always need to carry out calculations; they can compare expressions before they calculate.**

    Engage students in a general discussion about what it means for a number sentence to be true or false. Pose the following true/false problems (one at a time):

    $$12 - 9 = 3$$
    $$34 - 19 = 15$$
    $$5 + 7 = 11$$
    $$58 + 76 = 354$$

    Students explain how they know whether the number sentence is true or false. Students justify their solutions with their partner.
    Notice which students are calculating and which students are using relationships to determine whether the problems are true or false.

4. Pose the following true/false problem:

    $$27 + 48 - 48 = 27$$

    Students justify their answers.
    This problem establishes that students do not necessarily have to calculate to decide if a number sentence is true or false.

5. Ask students to see if they can figure out in their heads whether the following problem is true or false (without adding or subtracting):

$$48 + 63 - 62 = 49$$

Students justify their answer.
This problem extends the idea that was used in the previous problem.

6. Pose the following true/false problem:

$$174 + 56 - 59 = 171$$

Students justify their solutions.

This problem is slightly more complicated than the preceding problem because students have to recognize that 59 breaks apart to 56 + 3 and that they can subtract 56 from the 56 given in the problem, and then they have to subtract 3 more from 674.

7. **Goal 2: To use properties of numbers and operations to think about relations between numerical expressions.**

Review open number sentences. Pose the following problem:

What number would you put in the box to make this a true number sentence?

$$7 + 6 = \square + 5$$

Students justify their solutions.

8. Pose the following problems (one at a time):

$$33 + 18 = \square + 32$$
$$18 + 22 = 27 + \square$$
$$67 + 83 = \square + 82$$

Students justify their solutions.
Children must recognize that they can use relational thinking to solve these problems without carrying out all the calculations.

9. Up until this point, boxes have been used to represent an unknown in an open number sentence. Students readily adapt to using letters to represent variables and unknowns. Pose the following problem:

$$12 + 9 = 10 + 8 + c$$

What is the value of c?

Students justify their solutions.
If students justify their answers with an explanation focusing on computation, ask how this problem could be solved without adding 12 + 9 or 10 + 8 (e.g., 10 is two less than 12 and eight is one less than nine, so c must be 3).

10. Pose a problem with larger numbers but the same general structure, as follows:

$$345 + 576 = 342 + 574 + d$$

What is the value of d?

Students justify their solutions.

11. Pose the following problem:

$$46 + 28 = 27 + 50 - p$$

What is the value of p?

Students justify their solutions.

12. When students have figured out how to deal with addition problems, move to subtraction problems. Pose the following problem:

$$86 - 28 = 86 - 29 - g$$

What is the value of g?

13. **Goal 3: Using relational thinking to learn multiplication facts**

The following problems can be used to draw children's attention to relations among numbers that can make learning number facts easier.

- Knowing that addition and multiplication are commutative reduces the quantity of number facts that children have to learn by almost half.

True/False

$2 \times 4 = 4 \times 2$

What number would you put in the box to make this a true number sentence?

$4 \times 3 = 3 \times \square$

- Understanding the relation between addition and multiplication makes it possible for students to relate the learning of multiplication facts to their knowledge of addition.

True/False

$3 \times 7 = 7 + 7 + 7$
$3 \times 7 = 14 + 7$
$4 \times 5 = 10 + 10$
$6 \times 4 = 4 + 4 + 4 + 4$ (false)

- Focusing on specific relationships among multiplication facts can make it possible for students to build on the facts they have learned.

True/False

$3 \times 8 = 2 \times 8 + 8$
$6 \times 7 = 5 \times 7 + 7$
$8 \times 6 = 8 \times 5 + 6$ (false)
$7 \times 6 = 7 \times 5 + 7$
$9 \times 7 = 10 \times 7 - 7$

Sample problems to assist in developing relational thinking

True/False (not all are true)

$37 + 56 = 39 + 54$

$33 - 27 = 34 - 26$

$25 + 26 = 21 + 30$

$471 - 382 = 474 - 385$

$674 - 389 = 664 - 379$

$583 - 529 = 83 - 29$

Sample problems for developing understanding of the properties of numbers and operations within numerical expressions

73 + 56 = 71 + d	73 + 56 = 71 + 59 – d
68 + b = 57 + 69	68 + 58 = 57 + 69 – b
96 + 67 = 67 + p	96 + 67 = 67 + 93 + p
87 + 45 = y + 46	87 + 45 = 86 + 46 + t
92 – 57 = g – 56	92 – 57 = 94 – 56 + g
56 – 23 = f – 25	56 – 23 = 59 – 25 – s
74 – 37 = 75 – q	74 – 37 = 71 – 39 + q

Sample problems for developing base ten concepts

True/False (not all are true)

56 = 50 + 6	47 + 38 = 40 + 7 + 30 + 8
87 = 7 + 80	24 + 78 = 78 + 20 + 2 + 2
93 = 9 + 30	63 – 28 = 60 – 20 – 3 – 8
94 = 80 + 14	63 – 28 = 60 – 20 + 3 – 8
94 = 70 + 24	246 = 24 x 10 + 6

<u>Reference</u>

Carpenter, Thomas P. Franke, Megan Loef, Levi, Linda, <u>Thinking Mathematically: Integrating Arithmetic & Algebra in Elementary School</u>, Portsmouth, N.H.: Heinemann, 2003.