

Performance Assessment Task
Parallelogram Grade 7
This task challenges a student to use knowledge of measurement to identify and measure height in a triangle and to apply formulas to find area and perimeter of triangles and quadrilaterals. A student must use appropriate techniques, tools and formula to develop arguments regarding geometric ideas about perimeter.
Common Core State Standards Math - Content Standards
<p><u>Geometry</u> Draw, construct, and describe geometrical figures and describe the relationships between them. 7.G.2 Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measurements of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.</p> <p>Solve real-life and mathematical problems involving angle measure area, surface area, and volume. 7.G. 6 Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubs and right prisms.</p>
Common Core State Standards Math – Standards of Mathematical Practice
<p>MP.5 Use appropriate tools strategically. Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to indentify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.</p> <p>MP.6 Attend to precision. Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.</p>
Assessment Results
This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including

the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.

Grade Level	Year	Total Points	Core Points	% At Standard
7	2007	9	4	25%

Parallelogram

This problem gives you the chance to:

- use measurement to find the area and perimeter of shapes
-

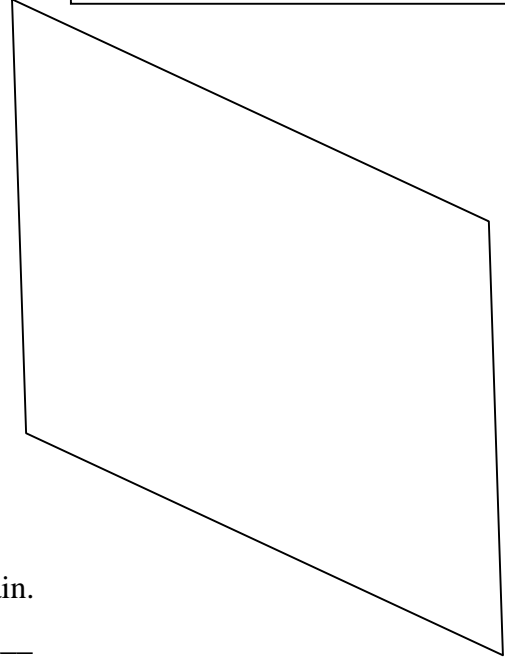
1. This parallelogram is drawn accurately.

Make any measurements you need, in centimeters, and calculate:

a. The area of the parallelogram. _____
Show your calculations.

b. The perimeter of the parallelogram. _____
Show your calculations.

The area of a parallelogram = base \times height

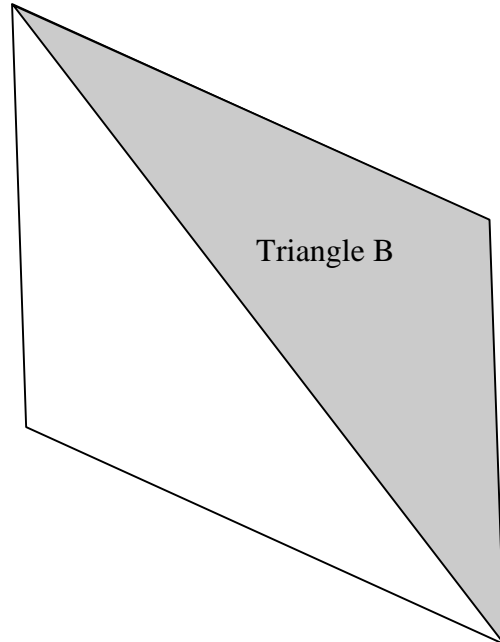
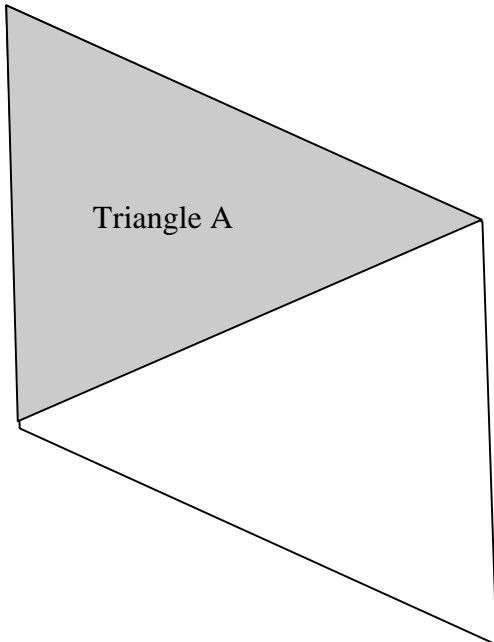


2. The diagram below shows the same parallelogram again.

a. Find the area of Triangle A. _____

b. Find the area of Triangle B. _____

c. Explain how you found your answers. _____



3. Which triangle has a larger perimeter, Triangle A or Triangle B?

Explain how you can tell without measuring.

4. Sketch a right triangle with the same area as Triangle A.

Your diagram does not need to be accurate.

Show how you figured it out.

Task 4: Parallelogram		Rubric	
<p>The core elements of performance required by this task are:</p> <ul style="list-style-type: none"> use measurement to find the area and perimeter of shapes <p>Based on these, credit for specific aspects of performance should be assigned as follows</p>		points	section points
1.a	<p>Gives correct answer in the range 33-39 square centimeters.</p> <p>Shows correct work such as: 7×5 or 6×6.</p> <p>Accept reasonable measurements shown on diagram.</p>	1 1	3
b	<p>Gives correct answer in the range 24-28 centimeters and shows work such as $2(6 + 7)$.</p> <p>Accept reasonable measurements shown on diagram.</p>	1	
2.a	Gives correct answer 17.5 square centimetres. Accept half of 1.a	1ft	3
b	Gives correct answer: 17.5 square centimetres. Accept half of 1.a	1ft	
c	Gives correct explanation such as: They are both equal to half the area of the parallelogram	1	
3.	Gives correct answer such as: Triangle B: both triangles have sides that match the two sides of the parallelogram. The third side of B is longer than the third side of A.	1	1
4.	<p>Sketches a correct triangle and shows correct work such as:</p> <p>The area of the triangle = $\frac{1}{2}$ base x height = 17.5.</p> <p>base x height = 35</p> <p>So if the base = 7 cm then the height = 5 cm</p>	2ft	2
<p>Note: Deduct 1 point for missing or incorrect units. (Need to show some evidence that area is measured in square units and that perimeter is a linear measure.</p>			
Total Points			9

Parallelogram

Work task and look at rubric. What is the mathematics being assessed? How are the big mathematical ideas different than when students are given dimensions?

A major piece of this task is understanding measurement issues. Look at student work for part 1 and 2. How many of your student measurements:

Omitted cm^2	Omitted cm	Omitted both labels	Measures off by distance from zero to end of ruler	Measures off by more than 0.5 cm	Numbers of unclear origin	No dimensions given

How often do students have opportunities to make their own measurements? What are some activities that you have done using measurement this year? Is measuring a frequent classroom process? What surprised you when looking at this data?

For part 1, How many of your students: multiplied the two sides together?

Correct calculation base times height	Measured a height	Multiplied the 2 sides together	Gave one of the dimensions as the area	Added the 2 sides	Multiplied 2 sides then multiplied or divided by 2	Other

Why do you think this was so difficult for students? How is the thinking for this task different from working problems in most textbooks? How has the cognitive demand changed?

For students who thought about the area of the triangles for part 2 as half the area from 1, part 2 was relatively easy. However, like the data in part 1, students for the most part did not draw in and measure a height. Look at student strategies for calculating in part 2. How many of your students:

Correct answer $(b \times h)/2$	Correct answer $1/2(\text{parall.})$	$(s_1 \times s_2)$ no division	$(s_1 \times s_2)/2$	$s_1 \times s_2 \times d$ $d = \text{diagonal}$	Measure of one side	$s_1 + s_2 + d$	Perimeter of parallelogram

Looking at these results, what types of experiences do students need to help them think about the big idea of area?

How do you help students make sense of how formulas are derived? Do you think students can make connections between the algebraic symbols and the concept of area?

Part 3 was difficult for students. They are not sure about all the parts needed to make a convincing argument. How many of your students:

- Mentioned that A and B had two common sides and the diagonal that formed that third side for B was longer, therefore perimeter of B was longer?
- Just said it looks bigger (referring either to A or B)?
- Its thinner, fatter, shorter, wider?
- Mentioned that the diagonal forming the two triangles is different, but did not state that the other two sides are the same?
- All 3 sides of B are bigger
- B has a larger area, so it has a larger perimeter
- Thought both triangles were the same because
 - They had the same area?
 - The perimeters of the parallelograms are the same
 - Both are half the parallelogram

How do you help students develop the logic of making convincing arguments? What are some examples of student discussions that allow them to compare and contrast different arguments so that they can explicitly describe qualities of a good argument or what's missing to make an argument more convincing?

Finally, look at student work for part 4. How many of your students:

Drew right triangle?	Drew a triangle?	Drew a shape other than a triangle?

Correct dimensions	No dimensions	One of the dimensions on the hypotenuse instead of the leg	Wrong dimensions to get area as that for 2a

What are your next steps for instruction?

What does this task make you think about what it means to understand an idea versus be able to calculate an answer?

Looking at Student Work on Parallelogram

Student A is able to draw in heights for the parallelogram and the triangles. The student makes a complete logical argument for comparing perimeters in part 3. The student makes a right triangle, labeling the dimensions of the legs. The student labels dimensions and calculations with appropriate units.

Student A

1. This parallelogram is drawn accurately.

Make any measurements you need, in centimeters, and calculate:

- a. The area of the parallelogram. 35cm²
Show your calculations.

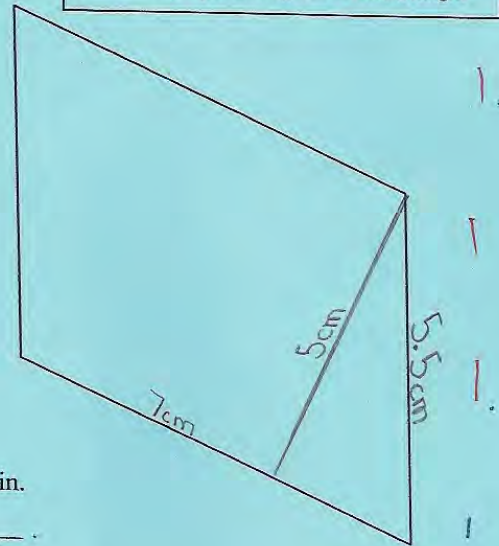
$$7 \cdot 5 = 35$$

- b. The perimeter of the parallelogram. 25cm
Show your calculations.

$$7 \cdot 2 = 14 \quad 14 + 11 = 25$$

$$5 + 5 \cdot 2 = 11$$

The area of a parallelogram = base \times height



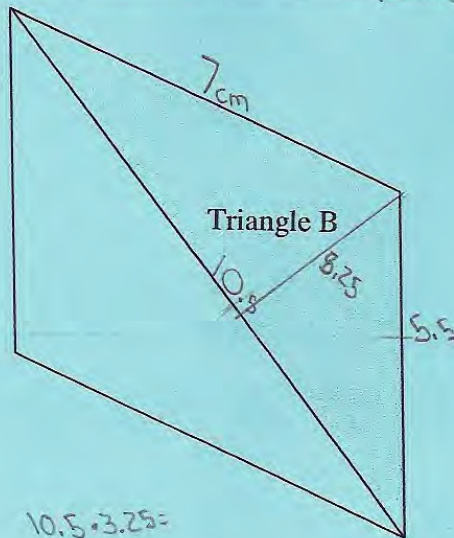
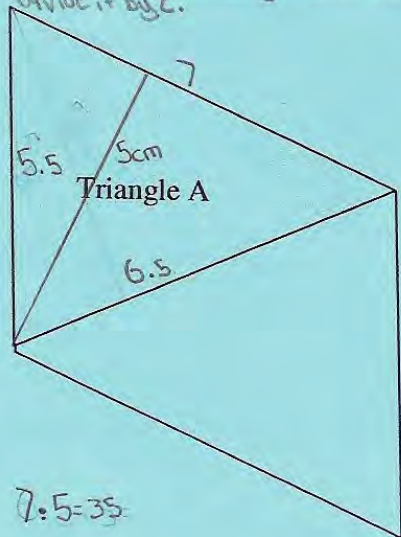
2. The diagram below shows the same parallelogram again.

- a. Find the area of Triangle A. 17.5cm²

- b. Find the area of Triangle B. 17.5cm²

- c. Explain how you found your answers. Both triangle A and triangle B are equal to

half of the parallelogram so all I had to do was find the area of the parallelogram and divide it by 2.



Student A, part 2

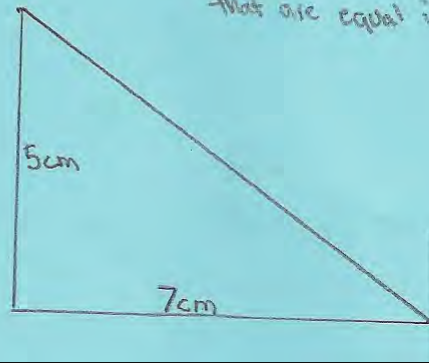
3. Which triangle has a larger perimeter, Triangle A or Triangle B?

Explain how you can tell without measuring.

Triangle B has a larger perimeter. I can tell because both Triangle A and B share 2 equal sides but they do not share their third sides. I can tell from eye that Triangle B's third side is much larger than Triangle A's third side.

4. Sketch a right triangle with the same area as Triangle A.
Your diagram does not need to be accurate.

Show how you figured it out.



Both my triangle and Triangle A have 2 sides that are equal in length.

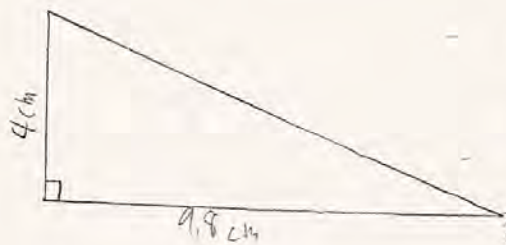
2.

Student B is the only student in the sample to find a correct set of dimensions without using measurements from previous shapes.

Student B

4. Sketch a right triangle with the same area as Triangle A.
Your diagram does not need to be accurate.

Show how you figured it out.



$$\begin{array}{r} \times 19.25 \checkmark \\ \hline 38.5 \end{array} \times 0 \quad 2$$

$$\begin{array}{r} 4 \overline{) 39.2} \\ \underline{36} \\ 25 \\ \underline{24} \\ 10 \end{array}$$

Student C makes a vertical side for the base of the parallelogram and uses appropriate measures. (Notice that the numbers are slightly smaller than expected. The student may have measured from the end of the ruler instead of from zero.) The student seems to be reconstructing the logic of the area formula each time, by showing how the shape can be decomposed and remade into a rectangle. This strategy helps the student design the right triangle correctly. Notes at the bottom of the part 2 seem to indicate that the student calculated the perimeter, but that doesn't help the student think out a convincing argument for why triangle B has a larger perimeter.

Student C

1. This parallelogram is drawn accurately.
 Make any measurements you need, in centimeters, and calculate:

a. The area of the parallelogram. 28.62 cm^2
 Show your calculations.
 $5.4 \times 5.3 \checkmark$

b. The perimeter of the parallelogram. 23.6 cm
 Show your calculations.
 $(5.3 \times 2) + (6.5 \times 2)$

The area of a parallelogram = base x height

2. The diagram below shows the same parallelogram again.

a. Find the area of Triangle A. $14.31 \text{ cm}^2 \checkmark$

b. Find the area of Triangle B. $14.31 \text{ cm}^2 \checkmark$

c. Explain how you found your answers. I found the height of the triangle, multiplied base x height and divided!

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Parallelogram Test 7

7.7

21.5

Student C, part 2

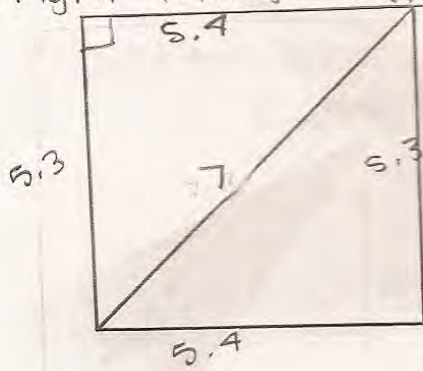
3. Which triangle has a larger perimeter, Triangle A or Triangle B?

Explain how you can tell without measuring.

Triangle B has a larger perimeter because it is longer but skinnier than Triangle A.

4. Sketch a right triangle with the same area as Triangle A.
Your diagram does not need to be accurate.

Show how you figured it out. I drew a square w/ the same area as the parallelogram, then made a right triangle out of it.



2 ft

$\sqrt{2}$

Student D is able to measure the correct height to calculate the area of the parallelogram and the area of triangle A. What error does Student D make in trying to draw a height in triangle B? Find 2 ways to redraw the height

Student D

1. This parallelogram is drawn accurately.

Make any measurements you need, in centimeters, and calculate:

- a. The area of the parallelogram. 35.96 cm²
Show your calculations.

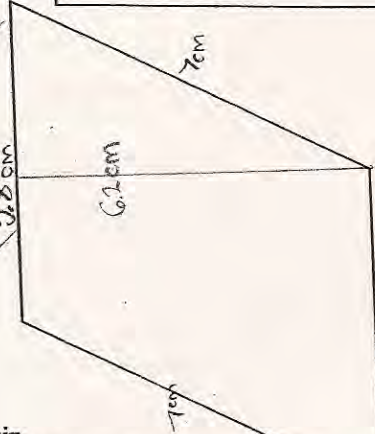
$$6.2 \times 5.8 = 35.96 \checkmark$$

- b. The perimeter of the parallelogram. 25.6 cm
Show your calculations.

$$7.7 + 5.8 = 13.5 \checkmark$$

$$13.5 + 11.6 = 25.1 \checkmark$$

The area of a parallelogram = base x height



2. The diagram below shows the same parallelogram again.

- a. Find the area of Triangle A. 17.05 cm²

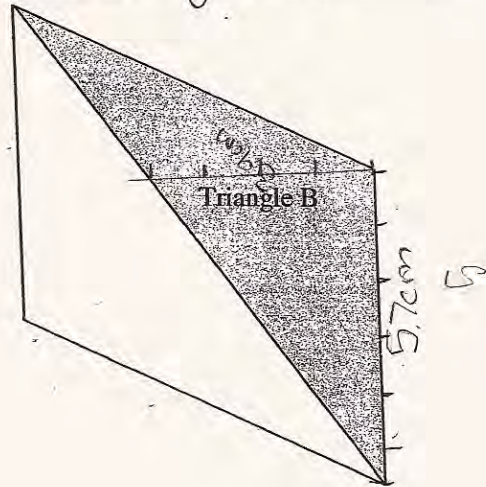
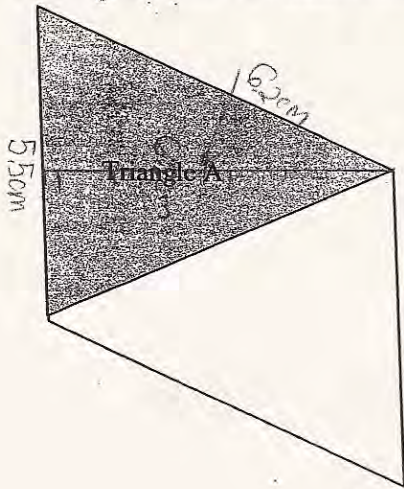
$$5.5 \times 6.2 = 34.1 \quad 34.1 \div 2 = 17.05 \checkmark$$

- b. Find the area of Triangle B. 11.15 cm²

$$5.7 \times 3.9 = 22.23 \quad 22.23 \div 2 = 11.115 \checkmark$$

- c. Explain how you found your answers. I found the height, multiplied.

Height x width, and then divided by 2. ✓



Student E uses a height to find the area in part 1. However the height measure is more than 0.5 cm off. Notice that **no** dimensions are drawn on any diagrams. The student appears to have multiplied base times height in part 2, but makes the common error of forgetting to divide by 2 as evidenced by the size of the numbers and formula.

Student E

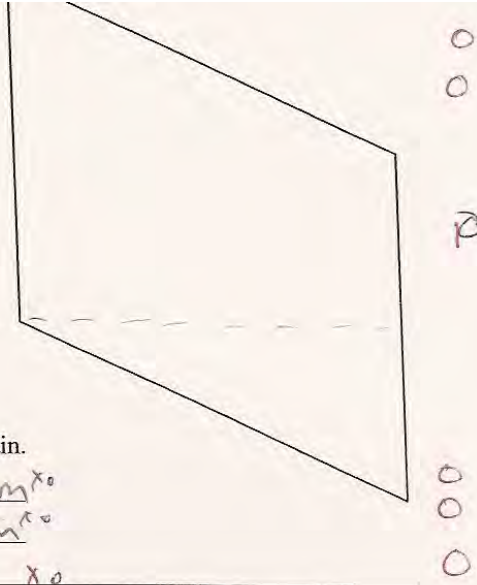
and calculate:

- a. The area of the parallelogram. $\times 27.5 \text{ cm}^2$
Show your calculations.

$$\begin{array}{r} 5.5 \\ \times 5 \\ \hline 27.5 \end{array} \times 0$$

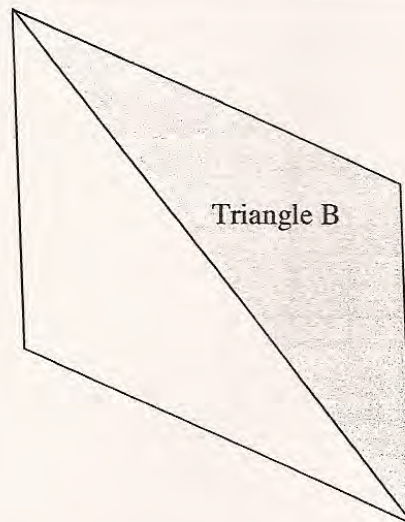
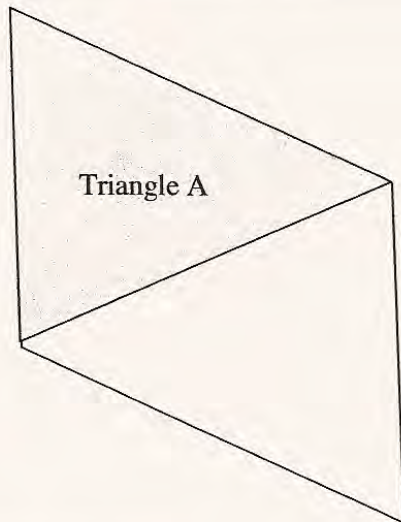
- b. The perimeter of the parallelogram. $\checkmark 23 \text{ cm}$
Show your calculations.

$$\begin{array}{r} 6.5 \\ + 6.5 \\ \hline 13 \\ \times 2 \\ \hline 26 \end{array} \text{ cm}$$



2. The diagram below shows the same parallelogram again.

- a. Find the area of Triangle A. $\times 27.5 \text{ cm}^2$
b. Find the area of Triangle B. $\times 25 \text{ cm}^2$
c. Explain how you found your answers. $B \cdot h \times 0$

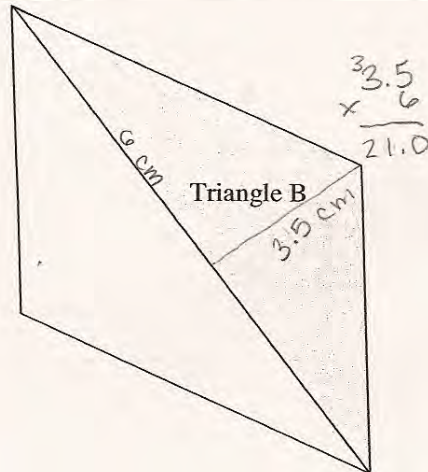
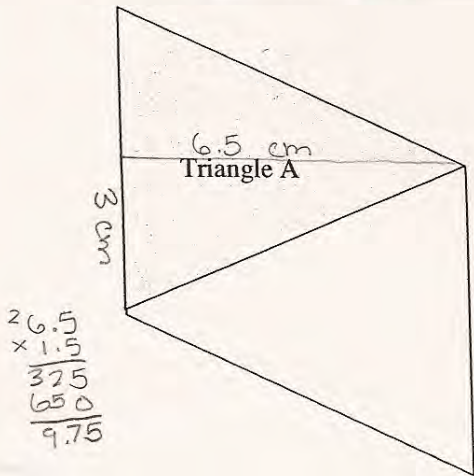


Student F does not draw a height to find the area of the parallelogram. However the student does draw in and measure heights for the triangles. The student only uses part of the base, instead of the entire base, to calculate the area of the triangles. *What assumption has the student made? Why doesn't it work, isn't the formula half the base times the height?*

Student F

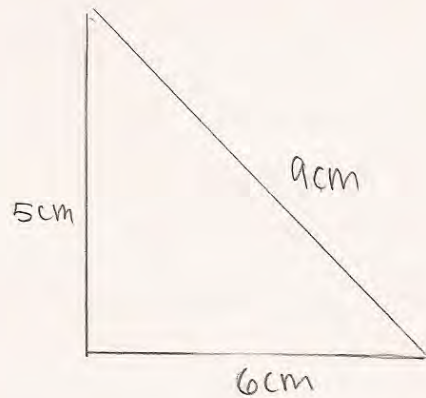
2. The diagram below shows the same parallelogram again.

- a. Find the area of Triangle A. 9.75 cm ✗
- b. Find the area of Triangle B. 21 cm ✗
- c. Explain how you found your answers. I split the triangle in half, then
 I did $bh\frac{1}{2}$ ✗



In many situations I can decompose part of a number and then recompose it to get an identical answer. Why doesn't this work for the sides of the triangle? Why isn't the area the same? Look at the work of Student G/

Student G



triangle A was $7\text{cm} + 7\text{cm} \times 6\text{cm}$
 so on this triangle it has $5\text{cm} \times 9\text{cm} \times 6\text{cm}$
 so if you take 3 cm from 9 and give 2 to 5 cm and 1 to 6 cm

9

Student H has decomposed the parallelogram into two triangles and a rectangle. Why aren't the calculations correct in part 1a? What did the student forget?

Student H

- This parallelogram is drawn accurately. Make any measurements you need, in centimeters, and calculate:

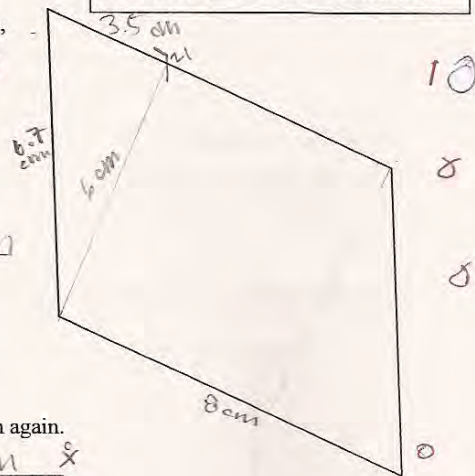
- The area of the parallelogram. Show your calculations.



- The perimeter of the parallelogram. Show your calculations.

174cm

The area of a parallelogram = base x height



- The diagram below shows the same parallelogram again.

- Find the area of Triangle A. 24.7cm
- Find the area of Triangle B. 38cm
- Explain how you found your answers. multiplied base x height then

divided by two.

Student I, like 5% of the students, has areas for the triangles in the hundreds. Notice the logic of the justification in part 3. Student I shows a typical drawing for part 4 of a triangle that is not a right triangle and has no dimensions.

Student I

1. This parallelogram is drawn accurately.

Make any measurements you need, in centimeters, and calculate:

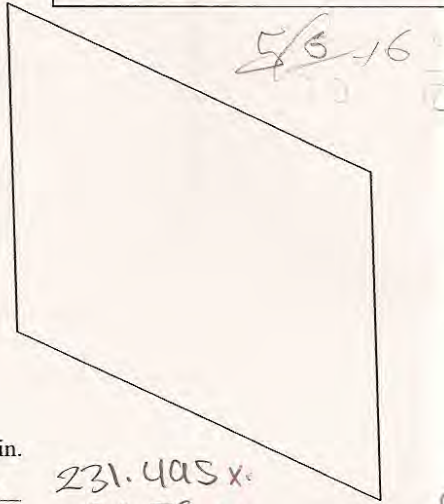
- a. The area of the parallelogram. 38.08 ✓
Show your calculations.

X^o

- b. The perimeter of the parallelogram. 24.8 X^o
Show your calculations.

X^o

The area of a parallelogram = base x height



2. The diagram below shows the same parallelogram again.

- a. Find the area of Triangle A. 39.05 231.405 X^o

- b. Find the area of Triangle B. 38.95 382.79 X^o

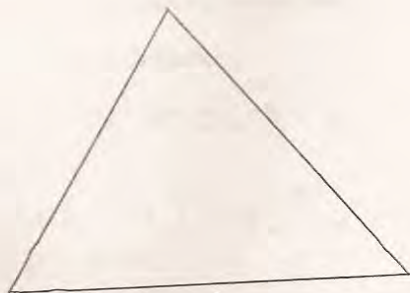
- c. Explain how you found your answers. I found out the sides and what their perimeter is and multiply them X^o



4. Sketch a right triangle with the same area as Triangle A. Your diagram does not need to be accurate.

Show how you figured it out.

I measured Triangle A and put the same area X^o.



Student J gives side lengths for area.

Student J

and calculate.

- a. The area of the parallelogram. $6\frac{1}{2}$
Show your calculations.

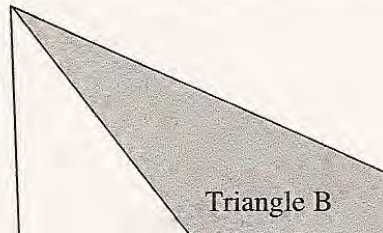
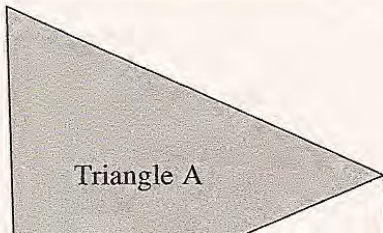
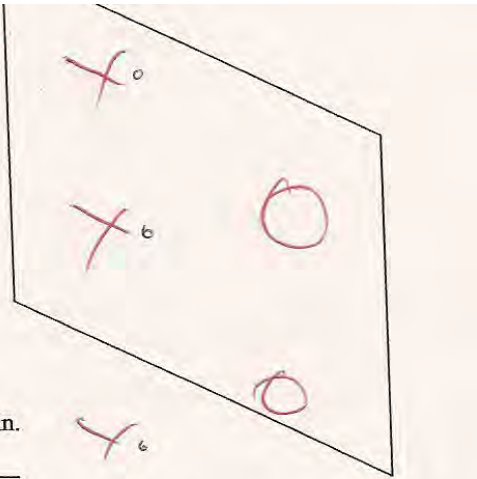
- b. The perimeter of the parallelogram. $20\frac{1}{2}$
Show your calculations.

2. The diagram below shows the same parallelogram again.

- a. Find the area of Triangle A. $6\frac{1}{2}$

- b. Find the area of Triangle B. $10\frac{1}{2}$

- c. Explain how you found your answers. I measured each side



Student K thinks shapes with the same area have the same perimeter.

Student K

3. Which triangle has a larger perimeter, Triangle A or Triangle B?

Explain how you can tell without measuring.

They have the exact same perimeter
because they are both half of a
triangle

While the primary purpose of this task is to look at student understanding about direct and indirect measurement, a rich task can also give insight into other misconceptions. The following set of students is having difficulty calculating with rational numbers. Look at the work of Student L. *What error has the student made in multiplying by a fraction? What is the misconception underlying this error?*

Student L

Make any measurements you need, in centimeters, and calculate:

a. The area of the parallelogram. 35 1/2 cm
Show your calculations.

b. The perimeter of the parallelogram. 7.5 cm
Show your calculations.

2. The diagram below shows the same parallelogram again.

a. Find the area of Triangle A.

What error has Student M made in multiplying decimals?

Student M

1. This parallelogram is drawn accurately.

Make any measurements you need, in centimeters, and calculate:

a. The area of the parallelogram. 3085
Show your calculations.

The area of a parallelogram = base × height

What error has Student N made in adding decimals?

Student N

b. The perimeter of the parallelogram. 13 cm
Show your calculations.

Look at the multiplication error in the work of Student O. How is it different from the misconception of Student M?

Student O

- b. The perimeter of the parallelogram. 24.2cm
Show your calculations.

$$\begin{array}{r} 8.5 \\ + 7.1 \\ \hline 15.5 \end{array} \quad \begin{array}{r} 7.1 \\ + 5.9 \\ \hline 13.0 \end{array} \quad \begin{array}{r} 10 \\ + 4 \\ \hline 14 \end{array}$$

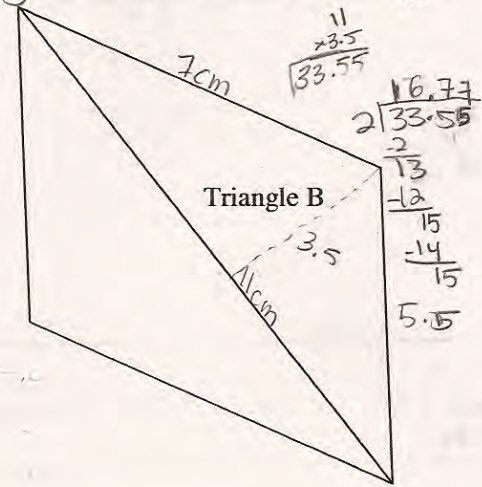
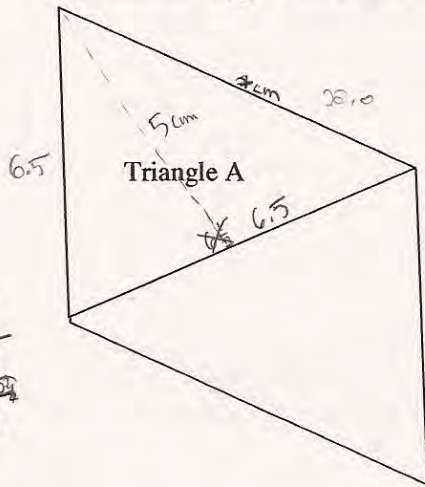
$$\begin{array}{r} 15.5 \\ + 8.7 \\ \hline 24.2 \end{array}$$

2. The diagram below shows the same parallelogram again.

- a. Find the area of Triangle A. 15.12cm²

- b. Find the area of Triangle B. 16.77cm²

c. Explain how you found your answers. I found the measurements of the triangles and the height. Times them then ÷ by 2.



<i>Student Task</i>	Use measurement to find the area and perimeter of shapes.
Core Idea 4 Geometry and Measurement	<p>Apply appropriate techniques, tools, and formulas to determine measurements.</p> <ul style="list-style-type: none"> Develop and critique inductive and deductive arguments concerning geometric ideas and relationships, such as congruence and similarity.

Mathematics in the task:

- Ability to measure using a centimeter ruler
- Ability to draw and measure an altitude or height in a parallelogram and a triangle
- Ability to calculate area of a parallelogram and triangle
- Ability to develop a logical argument on perimeter with statements about the relative sizes of all the sides
- Ability to draw a right triangle and to find and to label leg size needed to obtain a given area

Based on teacher observation, this is what seventh grade students knew and were able to do:

- Measure dimensions and put quantity on a drawing or diagram
- Calculate the perimeter of a parallelogram

Areas of difficulty for seventh graders:

- Understanding the difference between height and length of a side in a parallelogram and a triangle
- Finding the appropriate numbers to substitute into an area formula
- Dividing by two when finding the area of a triangle
- Comparing perimeters of two shapes without measurement, thinking about the relationships of the sides
- Drawing a right triangle
- Labeling the legs instead of the hypotenuse, to give the dimensions for a given area
- Applying appropriate measurement units to calculations (cm or cm²)

MARS Test Task 4 Frequency Distribution and Bar Graph, Grade 7

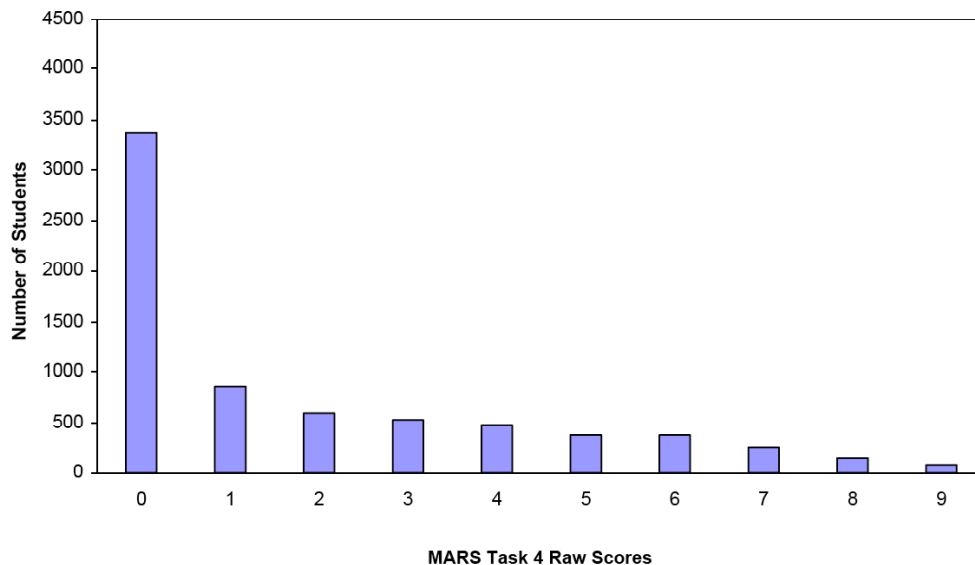
Task 4 - Parallelogram

Mean: 1.91 StdDev: 2.46

Table 38: Frequency Distribution of MARS Test Task 4, Grade 7

Task 4 Scores	Student Count	% at or below	% at or above
0	3372	47.7%	100.0%
1	846	59.7%	52.3%
2	586	68.0%	40.3%
3	532	75.5%	32.0%
4	480	82.3%	24.5%
5	371	87.5%	17.7%
6	370	92.8%	12.5%
7	263	96.5%	7.2%
8	159	98.7%	3.5%
9	90	100.0%	1.3%

Figure 47: Bar Graph of MARS Test Task 4 Raw Scores, Grade 7



The maximum score available for this task is 9 points.

The minimum score needed for a level 3 response, meeting standard, is 4 points.

About half the students, 52%, could find the perimeter of the parallelogram and describe a correct method of finding the area of a triangle but most did not label answers with correct units. About 25% of the students could find the perimeter, describe how to find the area of a triangle, compare the perimeter of two triangles, and draw a right triangle that with dimensions needed to get a certain area. Again most of these students did not label answers with correct units. Less than 2% of the students could meet all the demands of the task, including drawing and measuring heights of parallelograms and triangles, calculating areas of triangles and parallelograms, and labeling answers with appropriate units. 48% of the students scored no points on this task. 90% of students with that score attempted the task.

Par allelogram

Points	Understandings	Misunderstandings
0	90% of the students with this score attempted the task.	9% of the students gave a correct answer for perimeter with no calculations. 4% made decimal errors in adding the perimeter of the parallelogram. 4% had measuring errors for at least 1 side of the figure.
1	Students could measure sides in centimeters and calculate perimeter of a parallelogram. Students could also either explain a correct procedure for finding the area of a triangle or give a reason why triangle B had a larger perimeter than triangle A. They did not label units.	18% thought the formula for area of a triangle was length times width. 10% did not attempt to explain how to find the area of a triangle. 9% did not attempt to compare the perimeter of the two triangles. 9% talked about the perimeter of the parallelogram. 15% just said, "it looks bigger." 15% said all the sides of B were longer. 11% knew the diagonal was longer, but did not mention the other two sides. 4% gave arguments about its skinnier or fatter. 4% thought they would be the same, because they have the same area.
2	Students could find the area of the two triangles using the logic that each triangle was half the area of the parallelogram. (No labels and using an incorrect area from part 1)	26% of the students did not use cm^2 for area measures. 29% did not give units for area or perimeter. 15% gave measurements that were off by more than 0.5 cm. 4% did not show any dimensions on their diagrams.
4	Students could find the perimeter of parallelogram, explain how to find the area of a triangle, explain why B has a larger perimeter, and draw a right triangle with correct dimensions to get a certain area. They did not label units.	43% of the students did not draw a right triangle. 16% of the students did not attempt to draw a right triangle. 27% drew triangles that were not right triangles. 33% gave triangles with no dimensions. 10% drew shapes that were not triangles. 4% labeled one of the needed dimensions on the hypotenuse instead of the leg. 13% gave dimensions that would not give the needed area.
6	Students could find perimeter of a parallelogram, calculate area of triangles, explain how to find area of a triangle, and draw a right triangle with correct dimensions to match the area of triangle A. Students used unit labels.	Students had difficulty finding the area of the triangles. 25% of the students multiplied two of the sides together. 15% multiplied two sides together and divided by 2. 6% multiplied all 3 sides together. 4% just gave the length of one side for the area.

Points	Understandings	Misunderstandings
8	Students could find the height of the parallelogram and use it to calculate area, find the perimeter of shapes, find the area of triangles, and draw a right triangle that would have a given area. Students used unit labels.	Students could not compare the perimeters of the two triangles. Students, who didn't get this score, could not find the area of the parallelogram. 57% of the students multiplied two sides together. 6% just put the measurement of 1 side for the area. 5% added two sides together for area.
9	Students could find the height of the parallelogram and use it to calculate area, find the perimeter of shapes, compare perimeters, find the area of triangles, and draw a right triangle that would have a given area. Students used unit labels.	

Implications for Instruction

Students need more opportunities to investigate shapes and make their own measurements. The reasoning for finding height and understanding how it is measured is different from the reasoning used for substituting numbers into a formula. Students should understand the reasoning for how area formulas are derived. They should be able to think about how a parallelogram can be decomposed and recomposed to form a rectangle. This type of understanding helps students to understand why the measurement of the height is different than that of the side length. If possible, students should have the opportunity to investigate with dynamic geometry software programs how the decomposition works and compare with an overlapping picture of a rectangle with the height the same as the side length. Students should also have opportunities to investigate the relationships between a rectangle and a triangle. Again, using geometry software, like Geometer Sketchpad, to investigate these relationships would help students develop a better understanding of these relationships.

Students need to be exposed to a greater variety of models for triangles. They also need regular practice making their own diagrams, drawing in heights, and labeling needed measurement for helping them think about problems. Too many students did not draw right triangles in part 4.

Students also need more experience developing logical arguments about a variety of topics. Students should be thinking about comparing attributes between figures for similarity and be able to notice and to state that two of the sides of the triangles in part 2 are the same size. Having opportunities to see and compare different arguments in class and discuss which is more convincing helps students develop the logic needed to make mathematical arguments.

Ideas for Action Research – Confronting Issues of Measurement

Students need to have more opportunities to measure. Many students showed measurements that started from the end of the ruler, instead of zero. Others made measurements that were off by more than 0.5 cm. Students did not know what parts of the drawing to even measure. To plan a follow up lesson to using this task, pick some interesting pieces of student work that will help students confront and grapple with some of the major misconceptions. Make the misconceptions explicit and up for public debate.

During the discussion, it is important for students to notice and point out the errors in thinking.

Consider starting with a prompt, such as:

When I looked at different students diagrams I noticed that some students thought the dimensions were 7cm. by 5.5 cm. Others had diagrams showing dimensions of 5.3 cm by 6.5 cm or 5 cm. by 6 cm. What is a reasonable range for the measurements? What mistakes do you think students might be making?

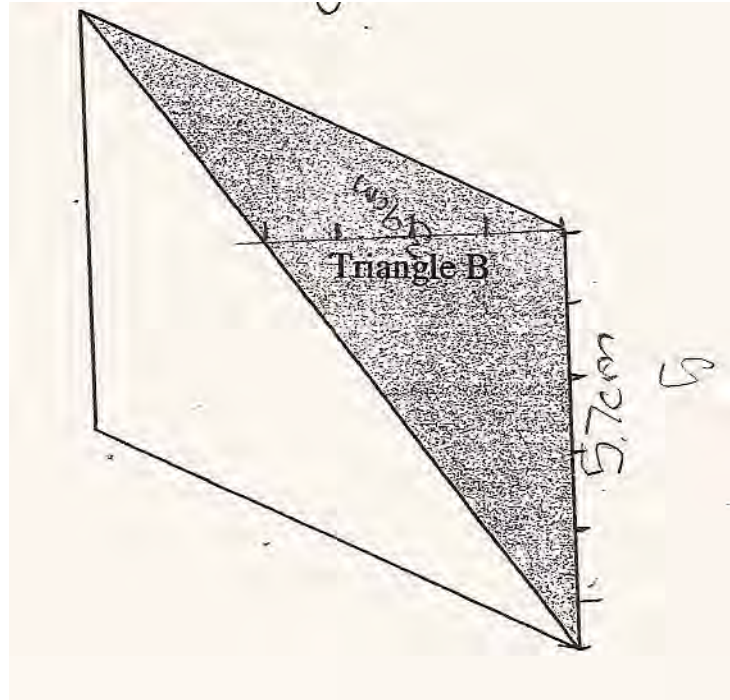
As students come to agreement about the measurements, put them on a diagram on the board. Change numbers in further prompts to match those agreed upon by the task. Now address the issue of area and height

Elizabeth says she thinking the area is 7 cm. times 5.5 cm. and gets an answer of 38.5 cm^2 . Francis disagrees. She says that Elizabeth didn't consider the height. What do you think she meant? How do we find the area of a parallelogram?

Hopefully this will prompt students to discuss why the side measurements are not the same as the height of the parallelogram. Now try to get students to discuss how to draw a height. If they don't think of it on their own, ask a question like:

Samuel multiplied 6.2 and 5.5 to get his area. What was he measuring? Will this work? Why or why not?

Martha has measured the base and the height of triangle B, but her answer is incorrect. What is the mistake that Martha has made? What would you tell Martha to help her with her diagram? The horizontal measure is 3.9.



Students should be able to find 2 different ways for measuring the base and height

Questions could also be posed about how to make a convincing argument.

Denise says, "Triangle A and B have the same perimeter because they are both half the parallelogram." Logan says, "Triangle B has a larger perimeter because it is longer but skinnier than Triangle A." Julia agrees. "I can just tell by looking that B is larger." Do you think these are convincing arguments? How could they be improved? What information do you have that could support your opinion?

Depending on class interest, you might want to pursue questions about how they made the right triangle for part 4 or you might want to plan some further lessons about geometric shapes.